

ANALYTICAL DESCRIPTION OF THE SOLID PHASE PARTICLE DISTRIBUTION IN A MECHANICALLY AGITATED SYSTEM*Marie SÝSOVÁ^a, Ivan FOŘT^b and Vladimír KUDRNA^a^a *Department of Chemical Engineering,**Prague Institute of Chemical Technology, 166 28 Prague 6 and*^b *Department of Chemical and Food Process Equipment Design,
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Received June 9th, 1987

Accepted August 26th, 1987

A mathematical model of the solid particle random motion in a turbulent liquid flow was developed which resulted in an unconventional form of diffusion equation. The diffusion equation derived describes quantitatively the local extremes of the solid phase concentrations in the charge mixed which were verified experimentally. The comparison of the experimental and calculated distribution of local concentrations in the system shows an adequate qualitative agreement. The system consisted of a cylindrical conical-bottomed vessel with baffles and cylindrical draft-tube. A six-blade impeller with inclined (at $\alpha = 45^\circ$) plane blades was used for agitating.

In profound studying some operations connected with mass transfer and when designing the flow agitated reactors, it is necessary to start from a more detailed knowledge of hydrodynamics of heterogeneous systems and not to consider only the critical frequency of revolution of impeller (FRI). The knowledge of local concentrations of the solid phase in different places of vessel brings important information, e.g., for calculating the impeller power input and/or for calculating the kinetics of operations connected with mass transfer between the solid particles and liquid.

Particularly, the effect of geometrical parameters has recently been studied experimentally and theoretically on the distribution of local concentrations of the solid particles in liquid. The complete description of the flow of two-phase system under turbulent regime is, however, practically impossible. The momentum balance is very complicated¹, and the introduction of simplifying assumptions can be problematic and limits the validity of the solution proposed. A simpler approach to describe the concentration field is a mass balance. Peňáz and co-workers², when making up the model of distribution of the solid particle concentration have started from the equation of continuity for the solid particles and from the knowledge of the particle characteristic given by their order of the velocity magnitude. The derivation of rela-

* Part LXXIV in the series Studies on Mixing; Part LXXIII Collect. Czech. Chem. Commun. 53, 957 (1988).

tion for the coefficient of eddy diffusivity was carried out by them on the basis of the assumption of existence of local homogeneous isotropic turbulence in the whole agitated system.

Ambegaonkar and co-workers³ proposed a model which is to simulate the motion of dispersed particles by the statistical Monte Carlo method. The given procedure is a modification of the De Souza method⁴ but unlike it depends on the experimental data. Řeháková and Rod⁵ proposed a model for calculating the suspension concentrations based on the idea of suspending the solid particles through the diffusion mechanism. The diffusion coefficient was considered constant within the whole volume of vessel. Dítl⁶ and Dítl, Thýn, and Rieger⁷ have reported the measurement of the solid phase local concentrations by means of a radiometric method. Kendzia and Baumann⁸ proposed an optical method for measuring the suspension local concentration in non-translucent vessels. The concentration fields obtained exhibit local extremes. To describe concentration profiles in mechanically agitated cylindrical reactors, Fajner and co-workers⁹ used the diffusion equation, too, but the value of diffusion coefficient was considered constant in the whole volume. In the papers by Kudrna and co-workers¹⁰ and by Sýsová¹¹, an attempt was made to describe the solid particle motion in an agitated incompressible liquid on the assumption that this motion may be considered as a diffusion Markov process. The procedure proposed makes it possible to describe local extremes in the solid phase concentrations in charge which have been found experimentally¹².

The aim of this work has been to propose and experimentally verify a theoretical model of the determination of the solid particle spatial distribution in a mechanically agitated liquid under turbulent regime of its flow. The model exploits the knowledge of field of the average and fluctuation components of velocity of the bulk phase (liquid) and is intended for low concentrations of the solid phase when the particles do not significantly impair the flow pattern in the system investigated — a cylindrical vessel with radial baffles at its wall and a cylindrical draft-tube.

THEORETICAL

In the paper by Kudrna and co-workers¹⁰, a new approach was proposed to the derivation of relation which makes it possible to describe the distribution of the solid particle concentrations in the system solid phase-liquid. The relation describing the motion of solid particle in agitated incompressible liquid was derived on the assumption that this motion may be considered as a diffusion Markov process.

The equation derived by Kudrna and co-workers¹⁰ can be written in the form

$$\frac{\partial c}{\partial t} + \mathbf{v}(\mathbf{r}) \nabla c - \Delta[D_p(\mathbf{r}) c] = 0, \quad (1)$$

where c is the solid particle concentration, \mathbf{v} the velocity of mixture, and D_p is a so-called coefficient of eddy diffusivity of the solid phase. Eq. (1) makes it possible to describe the time and spatial changes of low concentrations of the solid particles in agitated incompressible liquid under a general turbulent regime of flow. The solution of Eq. (1) requires thus the knowledge of the velocity field of the suspension and, moreover, the field of values of the eddy diffusivity of the solid phase.

Flow Velocity of Two-Phase System

For the direct calculation of the velocity field of two-phase system we have neither enough experimental nor literature data, and therefore the following assumption of interaction of solid particles and liquid has been introduced: The deterministic component of the two-phase system velocity may be considered as a sum of the velocity of liquid considered without the presence of solid particles and a quantity proportional to the deterministic force acting on the particles. The introduced coefficient of proportionality has anisotropic character, and is a function of density and particle size and FRI. The effect of fluctuation components of velocities manifests itself in the value of eddy diffusivity¹³.

The forces acting on the particles may be, in a simplified way, divided into a force acting in the vertical direction – gravity and in the radial direction – centrifugal and/or centripetal force. This assumption can be written in the form

$$\mathbf{v} = \mathbf{u} + k\mathbf{w}, \quad (2)$$

where \mathbf{u} is the velocity of the liquid phase without the presence of solid particles and \mathbf{w} is the velocity of solid particles in liquid at rest.

With regard to the geometry of the system considered (see Fig. 1) it is further assumed that the system is cylindrically symmetrical and Eq. (2) can be further rearranged into the form

$$\mathbf{v} = \mathbf{u} + k_r w_o(r) \mathbf{e}_r - k_z w_p \mathbf{e}_z, \quad (3)$$

where w_p is the particle settling velocity in the liquid at rest and $w_o(r)$ the particle centrifugal velocity. The data on the field of average velocities of liquid without the presence of solid particles were provided by the papers by Hošťálek^{14,15} in which authors dealt with geometrically similar system, and proposed model equations for calculating the field of liquid mean velocity which brought out the experimental data fairly well.

For low concentrations of suspensions, it is possible to neglect the effect of the other particles on the settling velocity of single particle. The literature data on the values of settling velocities in turbulent media are rather incomplete, e.g., Schwartz-

berg and Treybal¹⁶ reported that they reached 30–60% of the value in liquid at rest, Fajner and co-workers⁹ gave about 45%. Correlation coefficient k_z in that work was a parameter of the model proposed similarly as parameter k_r , which corrected the effect of turbulent regime of flow on centrifugal velocity.

Eddy Diffusivity

As it is reported by Levich¹⁷ in his book, the mass transfer by the turbulent flow of liquid can be characterized by the turbulent diffusion coefficient and is formally analogous to the mass transfer by molecular diffusion in gases. The coefficient of eddy diffusivity of liquid can be combined with the quantities characterizing the turbulent flow of liquid. Thus we have^{1,17–19}

$$D_t \sim \Delta \langle u \rangle \cdot L \approx \langle u'^2 \rangle^{1/2} \cdot L, \quad (4)$$

where $\Delta \langle u \rangle$ is the change of mean liquid velocity and is of the same order as the velocity of turbulent pulsations $\langle u'^2 \rangle^{1/2}$, L is the macroscopic scale of turbulent pulsations.

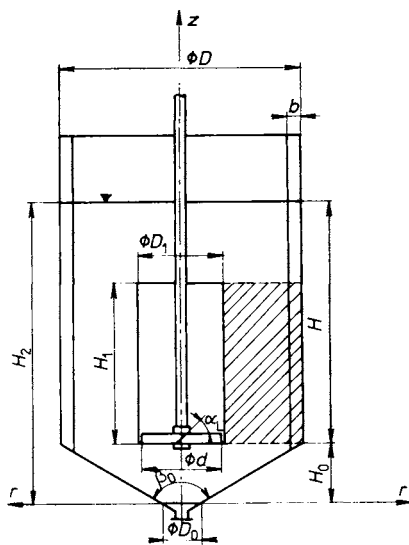


FIG. 1

Sketch of experimental equipment. (Hatched part represents the region of solution of diffusion equation.) $H_1 = 2/3H$, $b = 0.1D$, $D_1 = 1.1d$, $H = D$, $\beta_0 = 120^\circ$, $d/D = 1/3, 2/5$

For the Lagrange correlation, the turbulent macroscale takes the form²⁰

$$L_L = \langle u'^2 \rangle^{1/2} \int_0^T R_t dt, \quad (5)$$

and for the Euler description²⁰, the form

$$L_E = \int_0^\infty R_y dy. \quad (6)$$

As it is given by Komasa, Kuboi, and Otake¹⁸, we can write

$$D_f = \langle u'^2 \rangle^{1/2} \cdot L_L \sim \langle u'^2 \rangle^{1/2} \cdot L_E. \quad (7)$$

Analogously it is possible to deduce that eddy diffusivity of the solid phase is proportional to the fluctuation component of the particle velocity. Thus it holds

$$D_p \sim \langle v'^2 \rangle^{1/2} \cdot L_E. \quad (8)$$

For the ratio of diffusivities of the solid and liquid phases, Hinze²¹ and Soo¹ derived the relation

$$\frac{D_p}{D_f} = \frac{\langle v'^2 \rangle \int_0^t R_p(\tau) d\tau}{\langle u'^2 \rangle \int_0^t R(\tau) d\tau}. \quad (9)$$

Further, the limits²¹ hold

$$\lim_{t \rightarrow 0} \frac{D_p}{D_f} = \frac{\langle v'^2 \rangle}{\langle u'^2 \rangle}, \quad \text{and} \quad \lim_{t \rightarrow \infty} \frac{D_p}{D_f} = 1. \quad (10)$$

The value of ratio of eddy diffusivities determines the probability with which the solid phase particle follows the motion of liquid. The value of the fluctuation component of particle velocity can be found either experimentally (e.g., by the photographic method of traces) or by calculating as it is reported in literature^{1,13,21-24}. In this work we have chosen the method which is proposed by Lewins and Glastonbury²³, Soo^{1,13,24}, and Hinze²¹. It was justified by the knowledge of fluctuation component of liquid velocity and of some other characteristics (rate of energy dissipation) such as it is reported by Hořfálek^{14,15}.

Lewins and Glastonbury²³ derived, on the basis of the Tchen equation²⁵, for the motion of particle in oscillating liquid without acting gravity, an expression for the ratio of average fluctuation velocities of the solid and liquid phases in the form

$$\frac{\langle v'^2 \rangle}{\langle u'^2 \rangle} = b^2 + (1 - b^2) \frac{\sqrt{\pi}}{K} \exp \frac{1}{K^2} \operatorname{erfc} \left(\frac{1}{K} \right), \quad (11)$$

where

$$K = \sqrt{\pi/aT}, \quad a = 36\mu/(2\varrho_p + \varrho_f) d_p^2, \\ b = 3\varrho_f/(2\varrho_p + \varrho_f), \quad T = L_L/\langle u'^2 \rangle^{1/2}.$$

An analogous relation was published by Hinze²¹ and Soo^{1,24}.

Soo¹³ has introduced the relation for the ratio of eddy diffusivities

$$\frac{D_p}{D_f} \approx \frac{\langle v'^2 \rangle}{\langle u'^2 \rangle}. \quad (12)$$

For the eddy diffusivity of liquid, it is possible to derive, on the basis of literature data^{20,23}, the relation

$$D_f = \text{const.} \langle u'^2 \rangle^2 / \varepsilon, \quad (13)$$

where ε is the rate of dissipation of mechanical energy. Further we have assumed that the eddy diffusivity of solid particles is proportional to that of liquid

$$D_p = kD_f, \quad (14)$$

where the coefficient of proportionality k is approximately equal to the ratio of fluctuation components of velocities of the solid particle and liquid:

$$k \approx \langle v'^2 \rangle / \langle u'^2 \rangle. \quad (15)$$

The fluctuation components of the solid particle velocity were calculated by means of Eq. (11), the values of fluctuation component of the liquid velocity being taken from the results of experimental work¹⁴.

Theoretical Model

Let us assume further that the process is stationary. Considering that in the available literature there have been no data for calculating the eddy diffusivity of solid particles as a function of both coordinates (r and z), we consider only the dependence on the axial coordinate z . Moreover, with respect to the difficulties during the experimental investigation in the system examined, we accept the limitation of calculation of concentration field only to the region of cylindrical annulus (the hatched part in Fig. 1). With regard to the above facts and after introducing new dimensionless variables

$$R = \frac{r}{D/2}, \quad Z = \frac{z}{H_2}, \quad U_R = \frac{u_r}{Nd}, \quad U_Z = \frac{u_z}{Nd}, \quad W_p = \frac{w_p}{Nd},$$

$$W_o = \frac{w_o}{Nd}, \quad B = \frac{D_p}{NdD}, \quad (16a)$$

and, further, the dimensionless concentration C relating the local particle concentration to the bulk concentration of the solid phase in the whole vessel

$$C = c/c_v, \quad (16b)$$

we obtain the differential equation for steady state in the form

$$\begin{aligned} & [U_R(R, Z) + k_r W_o(R)] \frac{\partial C(R, Z)}{\partial R} + [U_Z(R, Z) - k_z W_p] \frac{\partial C(R, Z)}{\partial Z} - \\ & - \frac{1}{R} k_1 B(Z) \frac{\partial C(R, Z)}{\partial R} - k_1 C(R, Z) \frac{\partial^2 B(Z)}{\partial Z^2} - k_1 B(Z) \left[\frac{\partial^2 C(R, Z)}{\partial R^2} + \right. \\ & \left. + \frac{\partial^2 C(R, Z)}{\partial Z^2} \right] - 2k_1 \frac{\partial B}{\partial Z} \frac{\partial C(R, Z)}{\partial Z}, \quad (17) \end{aligned}$$

where k_r , k_z , and k_1 are parameters of the model whose optimum values are obtained from the comparison of experimental results and theoretical solution.

The solution of Eq. (17) in the examined region (hatched region in Fig. 1) delimited by the vessel wall,

$$R = 1, \quad Z \in \langle H_0/H_2, (H_1 + H_0)/H_2 \rangle, \quad (18a)$$

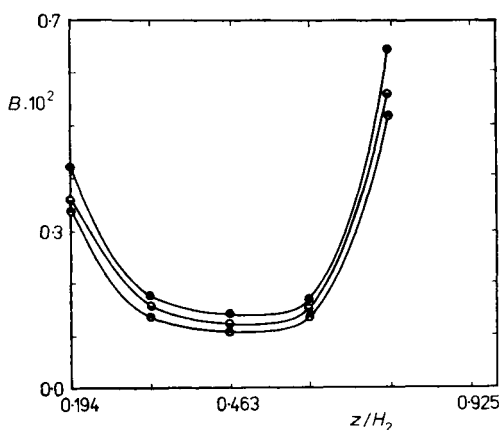


FIG. 2

Dependence of dimensionless eddy diffusivity B on axial coordinate. Charge no. 1: $d/D = 1/3$, water, $d_p = 0.45 \cdot 10^{-3}$ m, $c_v = 0.04$ m³/m³; ● $N = 4.12$ s⁻¹, ○ $N_K = 5.56$ s⁻¹, ● $N = 6.5$ s⁻¹

external wall of cylindrical draft-tube,

$$R = D_1/D, \quad Z \in \langle H_0/H_2, (H_1 + H_0)/H_2 \rangle, \quad (18b)$$

planes determined by bases of cylindrical externals,

$$Z = H_0/H_2, \quad R \in \langle D_1/D, 1 \rangle, \quad (18c)$$

and

$$Z = (H_0 + H_1)/H_2, \quad R \in \langle D_1/D, 1 \rangle, \quad (18d)$$

with boundary conditions for the draft-tube wall and/or vessel wall, written in the form

$$[U_R(R, Z) + k_r W_o(R)] C(R, Z) - k_1 B(Z) \frac{\partial C(R, Z)}{\partial R} = 0, \quad (19)$$

and finally, for planes defined by the bases of draft-tube, by conditions obtained from experimental investigations, gives the distribution of local concentrations of solid phase particles in the region delimited.

To check the correctness of calculation, we used the relation

$$\left(\frac{1}{2} - \frac{1}{3} \frac{H_0}{H_2}\right)^{-1} \int_{H_0/H_2}^{(H_0+H_1)/H_2} \int_{D_1/D}^1 C(R, Z) R \, dR \, dZ = I_v, \quad (20)$$

where I_v represents the relative amount of the solid particles suspended in the region delimited related to the whole amount of the solid particles in the system.

The available experimental data for calculating the field of eddy diffusivities (rate of energy dissipation ε and fluctuation velocity of the liquid phase $\langle u'^2 \rangle^{1/2}$) were rather limited¹⁴. Examples of the dependence of value of the dimensionless eddy diffusivity on the dimensionless axial coordinate are given in Fig. 2.

EXPERIMENTAL

The experiments were performed in a cylindrical vessel of diameter $D = 0.29$ m, with conical bottom of vertex angle $\beta_0 = 120^\circ$ and four radial baffles (Fig. 1). A cylindrical draft-tube was located co-axially in the vessel. A six-inclined-plane-blade impeller ($\alpha_L = 45^\circ$), which pumped liquid to the bottom when rotating, was used. The relative size of the impeller was $d/D = 1/3$ or $2/5$. The liquid charge was water and aqueous glycerol solution. The solid phase was formed by graded glass ballotini. Altogether 16 types of charges were measured (Table I). FRI changed from zero up to a value when air bubbles appeared in the charge.

The solid phase concentration was found by a calibrated conductivity sensor¹². Its value was read on a digital voltmeter and was averaged by means of a suitably chosen time constant. The concentration was measured in the vertical plane between the draft-tube and the vessel wall which went through the vessel center and halved the distance between two adjacent radial baffles, always in 24 places (network 3×8) under the same conditions of mixing. Examples of results of the experiments performed are illustrated in the form of isoconcentration curves in Figs 3 and 4.

RESULTS AND DISCUSSION

The experimental results have been reported in detail in literature^{11,12}, the analysis of the effect of geometrical and physico-chemical parameters on the distribution of the solid phase concentrations and on the value of FRI is given in the paper by Fořt and co-workers¹².

To describe the results, Eq. (17) with boundary conditions (19) was used. For single types of charges (altogether 16 types), three values of FRI (subcritical, critical, and supercritical -- see Table II) were chosen. To solve Eq. (17), the network method was used, and the given system of equations was solved numerically by means of the

TABLE I
Physico-chemical properties of model charges

Charge	d/D	c_v m^3/m^3	d_p $10^{-3} m$	ρ_p kg/m^3	ρ_f kg/m^3	ν $10^{-6} m^2/s$
1 ^a	1/3	0.04	0.450	2.639	1.000	1.000
2 ^a	1/3	0.08	0.450	2.639	1.000	1.000
3 ^a	1/3	0.04	0.925	2.654	1.000	1.000
4 ^a	1/3	0.08	0.925	2.654	1.000	1.000
5 ^a	2/5	0.04	0.450	2.639	1.000	1.000
6 ^a	2/5	0.08	0.450	2.639	1.000	1.000
7 ^a	2/5	0.04	0.925	2.654	1.000	1.000
8 ^a	2/5	0.08	0.925	2.654	1.000	1.000
9 ^b	1/3	0.04	0.450	2.639	1.118	4.001
10 ^b	1/3	0.08	0.450	2.639	1.118	4.001
11 ^b	1/3	0.04	0.925	2.654	1.113	3.902
12 ^b	1/3	0.08	0.925	2.654	1.113	3.902
13 ^b	2/5	0.04	0.450	2.639	1.110	3.548
14 ^b	2/5	0.08	0.450	2.639	1.110	3.548
15 ^b	2/5	0.04	0.925	2.654	1.110	3.548
16 ^b	2/5	0.08	0.925	2.654	1.110	3.548

^a Liquid phase is water. ^b Liquid phase is aqueous glycerol solution.

TABLE II

Values of integral I_v for experimental and calculated data and optimum values of model parameters k_1 , k_r , k_z

Charge	N, s^{-1}	$I_{v,exp}$	$I_{v,calc}$	k_1	k_r	k_z
1	4.12	0.073	0.083	1.0	0.3	0.3
	5.56	0.173	0.203	1.0	0.3	0.3
	6.50	0.180	0.217	1.0	0.3	0.3
2	5.99	0.141	0.188	1.0	0.3	0.3
	6.80	0.218	0.244	1.0	0.3	0.3
	7.14	0.230	0.256	1.0	0.3	0.3
3	6.00	0.114	0.276	1.0	0.3	0.3
	7.00	0.184	0.289	1.0	0.3	0.3
	8.00	0.194	0.376	1.0	0.3	0.3
4	9.00	0.210	0.447	1.0	0.3	0.3
	9.75	0.244	0.446	1.0	0.3	0.3
	10.50	0.246	0.377	1.0	0.3	0.3
5	4.00	0.100	0.126	1.0	1.0	1.0
	4.50	0.146	0.160	1.0	1.0	1.0
	5.00	0.142	0.151	1.0	1.0	1.0
6	4.50	0.152	0.149	1.0	1.0	1.0
	6.00	0.188	0.176	1.0	1.0	1.0
	7.50	0.180	0.174	1.0	0.3	0.3
7	4.00	0.059	0.071	0.3	0.2	0.3
	4.50	0.093	0.124	0.3	0.2	0.3
	5.00	0.104	0.128	0.3	0.2	0.3
8	5.50	0.111	0.169	0.3	0.2	0.2
	6.50	0.167	0.215	0.3	0.2	0.2
	7.50	0.195	0.182	0.3	0.2	0.2
9	3.50	0.127	0.152	1.0	0.3	1.0
	4.50	0.254	0.298	0.3	0.3	0.3
	6.50	0.320	0.348	0.3	0.3	0.3
10	5.00	0.230	0.256	0.3	0.3	1.0
	6.00	0.296	0.324	0.3	0.3	1.0
	7.50	0.305	0.343	0.3	0.3	1.0
11	5.00	0.173	0.380	0.3	0.3	0.3
	6.00	0.308	0.525	0.3	0.3	0.3
	7.50	0.361	0.485	0.3	0.3	0.3
12	6.00	0.171	0.144	0.3	0.3	0.3
	7.00	0.239	0.222	0.3	0.3	0.3
	8.50	0.250	0.246	0.3	0.3	0.3

TABLE II
(Continued)

Charge	N, s^{-1}	$I_{v,exp}$	$I_{v,calc}$	k_1	k_r	k_z
13	3.00	0.213	0.176	1.0	1.0	1.0
	4.00	0.252	0.209	0.3	1.0	1.0
	5.50	0.238	0.241	0.3	1.0	1.0
14	3.50	0.184	0.185	0.3	0.3	1.0
	4.00	0.256	0.256	0.3	0.3	1.0
	5.50	0.247	0.272	0.3	1.0	1.0
15	4.00	0.100	0.389	0.3	0.3	0.3
	4.50	0.170	0.252	0.3	0.3	0.3
	6.00	0.209	0.203	0.3	0.3	0.3
16	3.50	0.151	0.316	1.0	0.3	0.3
	4.50	0.219	0.365	1.0	0.3	0.3
	6.00	0.302	0.355	1.0	0.3	0.3

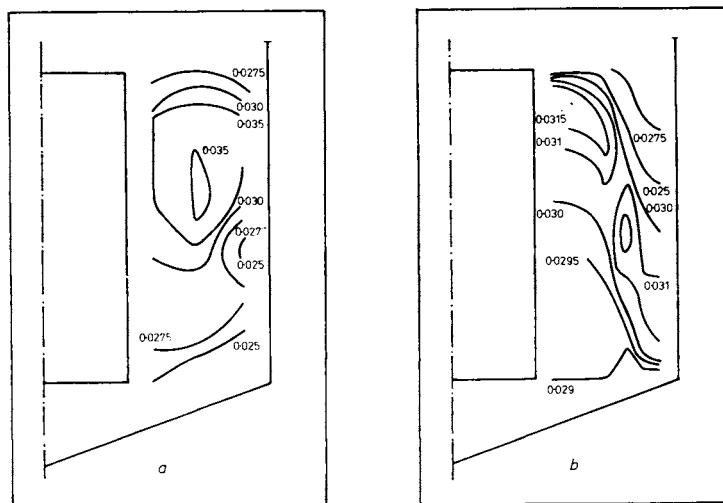


FIG. 3

Isoconcentration curves. *a* Experiment: (Charge no. 9: $d/D = 1/3$, glycerol, $d_p = 0.45 \cdot 10^{-3}$ m, $c_v = 0.04$ m³/m³, $N = 3.5$ s⁻¹); *b* calculation: $(k_1, k_r, k_z) = (1.0, 0.3, 1.0)$

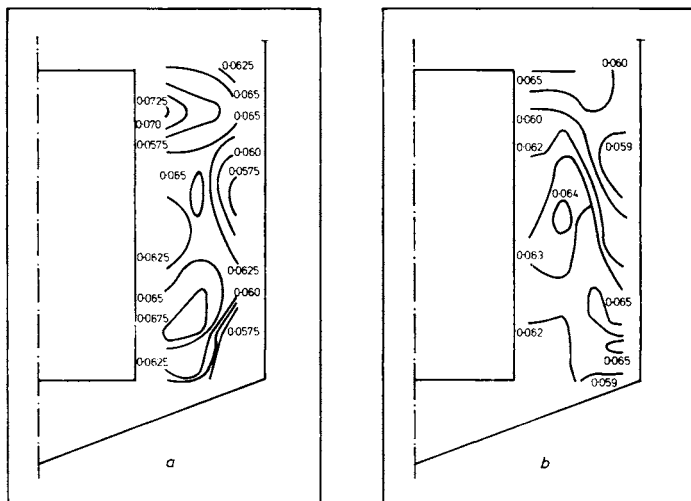


FIG. 4

Isoconcentration curves. *a* Experiment: (Charge no. 13: $d/D = 1/3$, glycerol, $d_p = 0.45 \cdot 10^{-3}$ m, $c_v = 0.04$ m³/m³, $N = 5.5$ s⁻¹); *b* calculation: $(k_1, k_r, k_z) = (0.3, 1.0, 1.0)$

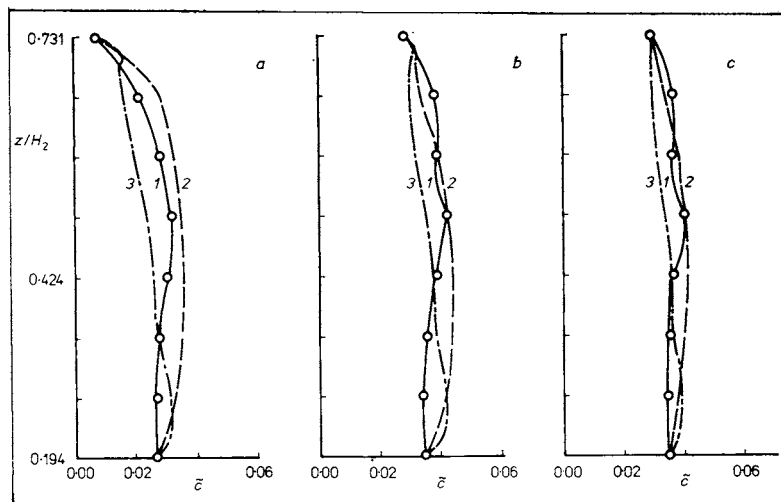


FIG. 5

Dependence of concentration c averaged over radial cross section on dimensional axial coordinate. Charge no. 5: $d/D = 2/5$, water, $d_p = 0.45 \cdot 10^{-3}$ m, $c_v = 0.04$ m³/m³. 1 Experiment; 2 calculation $(k_1, k_r, k_z) = (1.0, 1.0, 1.0)$; 3 $(1.0, 0.3, 0.3)$. *a* $N = 4.0$ s⁻¹, *b* $N_K = 4.5$ s⁻¹, *c* $N = 5.0$ s⁻¹

finite Gauss–Jordan method²⁶. Examples of the calculations carried out in this way are illustrated, in the form of isoconcentration curves, in Figs 3 and 4.

The obtained results (experimental and calculated) are moreover presented in the form of dependence of concentration (averaged over radial section) \bar{c} on dimensionless axial coordinate Z . Examples of these dependences for both the cases of system sizes ($d/D = 1/3$ and $2/5$) are plotted in Figs 5–9.

Further we have calculated, by means of Eq. (20) for each case measured, the relative amount of suspended solid particles in the region measured related to the total amount of solid particles in system (quantity I_v in Table II). The parametric sensitivity of the model was also evaluated. The most convenient parameters of the model (k_r, k_z, k_1) are as well given in Table II as obtained from an empirical comparison of the calculated concentration fields from the values found experimentally.

It was found (see Figs 5–9) that owing to an increase in FRI, the calculated distribution of local concentrations approaches to the concentration fields found experimentally (more homogeneous particle dispersion in charge). The lowest deviations of theoretical predictions from experimental results exhibit glycerol charges with particles of smaller diameter and low average volumetric concentration (Figs 3, 4 and 7, 8) where comparatively good agreement with experimental distribution

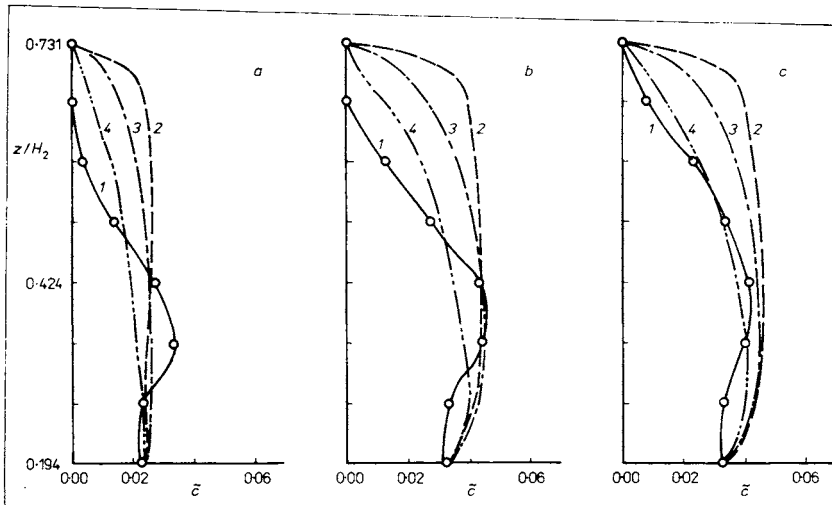


FIG. 6

Dependence of concentration \bar{c} averaged over radial cross section on dimensionless axial coordinate. Charge no. 7: $d/D = 2/5$, water, $d_p = 0.925 \cdot 10^{-3}$ m, $c_v = 0.04$ m³/m³. 1 Experiment; 2 calculation (k_1, k_r, k_z) = (1.0, 1.0, 1.0); 3 (1.0, 0.3, 0.3); 4 (0.3, 0.2, 0.3). a $N = 4.0$ s⁻¹, b $N_K = 4.5$ s⁻¹, c $N = 5.0$ s⁻¹

of the solid phase was reached. Significant effect on the solid particle distribution in charge showed viscosity of charge and size of solid particles; the average volumetric concentration in the given range did not manifest itself as a significant parameter. The calculated concentration field and the dependence of quantity \bar{c} on axial coordinate for the particles with diameter $d_p = 0.45 \cdot 10^{-3}$ m do not show great deviations from experiment, whereas the particles with diameter $d_p = 0.925 \cdot 10^{-3}$ m exhibit increasing deviations with increasing ratio z/H_2 (see Figs 9, 6). These deviations can be probably accounted for by a great value of centrifugal velocity (parameter k_r) compared to reality, which is caused by neglecting the interactions of particles. For charges with higher viscosity, a better agreement with experimental results was attained because these charges were mixed up already at lower values of FRI. These trends can be found out even on the curves $\bar{c} = f(Z/H_2)$ (see Figs 7–9). Also the value of quantity I_v (see Table II) shows that the highest deviations between the value obtained from experimental data and from calculated ones occur with charges which contain particles with higher value of diameter d_p .

From the comparison of experimental data and calculated isoconcentration curves (Figs 3, 4) can be judged that the proposed description exhibits above all qualitative agreement with experimental results. Only such an agreement is caused,

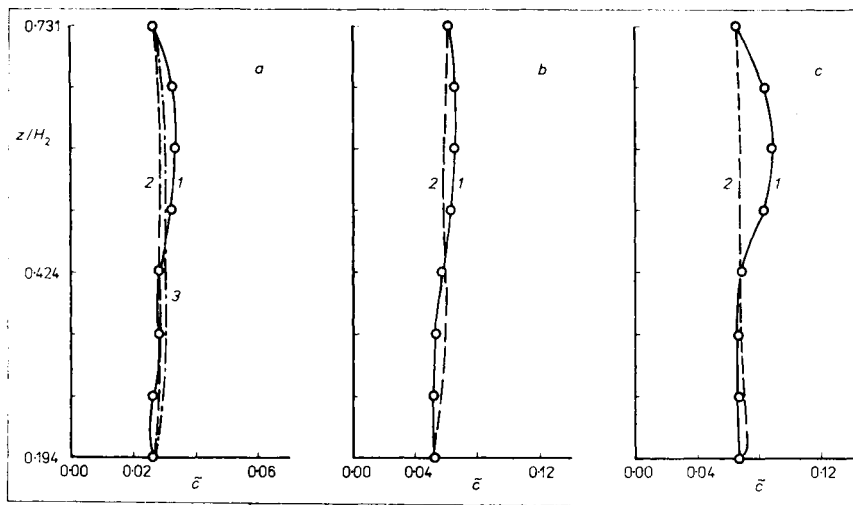


FIG. 7

Dependence of concentration \bar{c} averaged over radial cross section on dimensionless axial coordinate. Charge no. 9: $d/D = 1/3$, glycerol, $d_p = 0.45 \cdot 10^{-3}$ m, $c_v = 0.04 \text{ m}^3/\text{m}^3$. 1 Experiment; 2 Calculation (k_1, k_r, k_2) = (0.3, 0.3, 0.3); 3 (1.0, 0.3, 1.0). a $N = 3.5 \text{ s}^{-1}$, b $N_K = 4.5 \text{ s}^{-1}$, c $N = 6.5 \text{ s}^{-1}$

apart from the influence of experimental errors, above all by applying the kinematic model¹⁰ to the process in which gravity and centrifugal forces play a significant role. Moreover, by knowing (for the region studied) only the dependence $D_p = D_p(z)$ and not the more complex dependence $D_p = D_p(z, r)$. Substantially more accurate solution of the problem would be probably possible to reach by direct applying of stochastic differential equations¹⁰ describing the motion of solid particle in liquid and by solving the respective Kolgomorov equation¹⁰. This procedure would make it possible to express more concretely the effect of forces acting on particles but would lead to substantially more complicated expressions. From this point of view the procedure proposed here is a compromise. The model proposed (see Eq. (17)) made it possible to describe local extremes of concentration field which were found experimentally (Figs 3, 4). On using the classical diffusion equation (e.g., Peňáz and co-workers²), the extremes failed to be calculated.

From the study of the parametric sensitivity of the proposed theoretical model, the following conclusions followed: For the charges formed by water and particles of diameter $d_p = 0.45 \cdot 10^{-3}$ m, as significant parameters have appeared those which correct the centrifugal and settling velocities (k_r, k_z). For charges with particles of diameter $d_p = 0.925 \cdot 10^{-3}$ m, all the parameters have appeared to be significant.

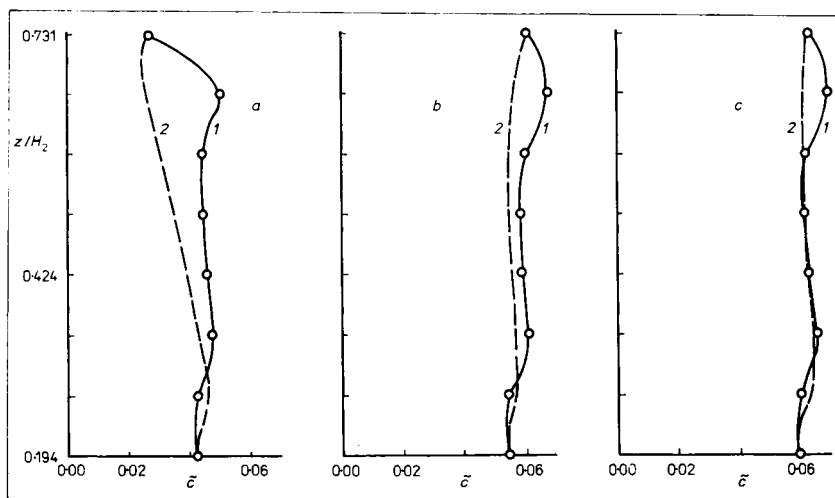


FIG. 8

Dependence of concentration \tilde{c} averaged over radial cross section on dimensionless axial coordinate. Charge no. 13: $d/D = 2/5$, glycerol, $d_p = 0.45 \cdot 10^{-3}$ m, $c_v = 0.04 \text{ m}^3/\text{m}^3$. 1 Experiment; 2 calculation; a $N = 3.0 \text{ s}^{-1}$ (k_1, k_r, k_z) = (1.0, 1.0, 1.0), b $N_K = 4.0 \text{ s}^{-1}$ (k_1, k_r, k_z) = (0.3, 1.0, 1.0), c $N = 5.5 \text{ s}^{-1}$ (k_1, k_r, k_z) = (0.3, 1.0, 1.0)

The greatest deviations were found for those cases when the solid particles do not reach the upper edge of draft-tube. The value of average volumetric concentration has not appeared in the given range as a significant quantity. For the charges formed by aqueous glycerol solution and particles of diameter $d_p = 0.45 \cdot 10^{-3}$ m, the greatest effect has shown the correction factor k_1 for eddy diffusivity and in most cases also the correction factor for centrifugal velocity k_r . For these charges, comparatively good agreement for all the concentration fields has been reached. For charges with particles of diameter $d_p = 0.925 \cdot 10^{-3}$ m, again all three parameters (k_1, k_r, k_z) have been found significant.

The proposed model equation, unlike the usually used diffusion equation, has made it possible to describe local extremes of the solid phase concentrations in agitated charge. The model was verified by the results of measurements of distribution of local concentrations of the solid particles in liquid. The comparison of the measured and calculated data shows an adequate agreement for the dependence of concentrations averaged with regard to the cross section on the axial coordinate. From the comparison of shapes of isoconcentration curves in the region examined, a qualitative agreement between the compared courses of dependences $c = c(r, z)$ was found. The agreement would be probably better for more detailed knowledge

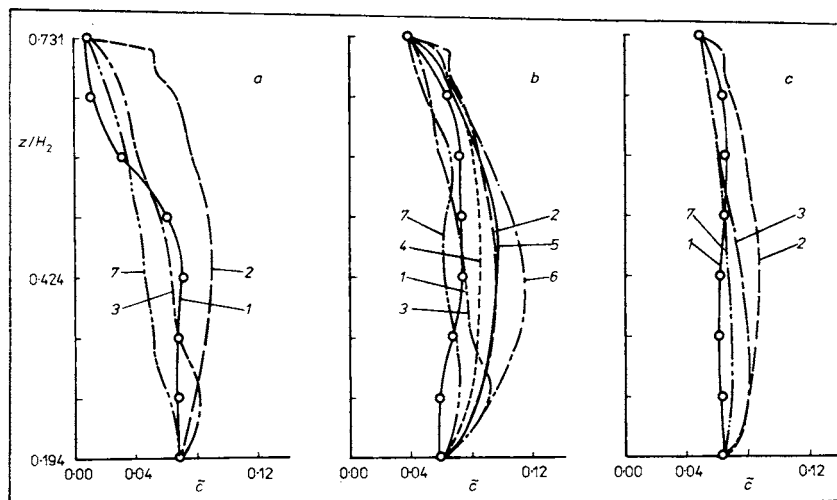


FIG. 9

Dependence of concentration \tilde{c} averaged over radial cross section on dimensionless axial coordinate. Charge no. 15: $d/D = 2/5$, glycerol, $d_p = 0.925 \cdot 10^{-3}$ m, $c_v = 0.04 \text{ m}^3/\text{m}^3$. 1 Experiment; 2 calculation (k_1, k_r, k_z) = (1.0, 1.0, 1.0); 3 (1.0, 0.3, 0.3); 4 (0.3, 1.0, 1.0); 5 (1.0, 0.5, 1.0); 6 (2.0, 1.0, 1.0) 7 (0.3, 0.3, 0.3). a $N = 4.0 \text{ s}^{-1}$, b $N_k = 4.5 \text{ s}^{-1}$, c $N = 6.0 \text{ s}^{-1}$

of turbulent characteristics of the region examined, expressed as the dependence of coefficients of eddy diffusivity D_p on coordinates r and z . In the end it is necessary to consider that the use of diffusion equation in usual form would in no case make it possible to calculate the extremes of local concentrations of the solid phase in liquid.

LIST OF SYMBOLS

a	parameter, s^{-1}
b	parameter
B	dimensionless coefficient of eddy diffusivity
c	local concentration, $m^3 m^{-3}$
c_v	average volumetric concentration, $m^3 m^{-3}$
C	dimensionless local concentration
d	impeller diameter, m
d_p	solid particle diameter, m
D	vessel diameter, m
D_f or D_p	coefficient of eddy diffusivity for liquid or solid phase, $m^2 s^{-1}$
D_1	draft-tube diameter, m
e_r, e_z	vectors of orthonormed coordinate base
H	height of liquid in cylindrical part of vessel, m
H_0	height of conical bottom, m
H_1	height of draft-tube, m
H_2	total height of liquid in vessel, m
I_v	relative amount of suspended solid particles
k	parameter
k_1, k_r, k_z	model parameters
K	parameter
K	coefficient
L	macroscale of turbulence, m
N	frequency of impeller revolution FRI, s^{-1}
r	radial coordinate, m
r	radius vector, m
R	dimensionless radial coordinate
R_L, R_p	correlation coefficient
R_y, R	correlation coefficient
t	time, s
T	time macroscale, s
u	velocity of liquid, $m s^{-1}$
$\langle u'^2 \rangle^{1/2}$	fluctuation component of liquid velocity, $m s^{-1}$
U	dimensionless liquid velocity
v	velocity of solid particle, $m s^{-1}$
$\langle v'^2 \rangle^{1/2}$	fluctuation component of solid particle velocity, $m s^{-1}$
w	velocity of motion of single particle in liquid at rest, $m s^{-1}$
w_0	centrifugal velocity, $m s^{-1}$
w_p	settling velocity, $m s^{-1}$
W_p	dimensionless settling velocity

W_0	dimensionless centrifugal velocity
y	coordinate, m
z	axial coordinate, m
Z	dimensionless axial coordinate
α_L	angle of blade inclination
β_0	vertex angle of conical bottom
ε	rate of energy dissipation per mass unit, $\text{m}^2 \text{s}^{-3}$
μ	dynamic viscosity, Pa s
ν	kinematic viscosity, $\text{m}^2 \text{s}^{-1}$
ρ	density, kg m^{-3}
τ	time, s
φ	tangential coordinate

Subscripts and Superscripts

r	radial
z	axial
p	referred to solid phase
f	referred to liquid
~	averaged over radial cross section
'	referred to solid phase
E	Euler correlation
L	Lagrange correlation
K	critical value of FRI (when reaching suspension of solid phase)

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Translated by J. Linek.